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A note on the use of a 2 x 2 matrix operating on linear colour-difference signals

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Research Department, Engineering Division
THE BRITISH BROADCASTING CORPORATION

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SUMMARY

The use of 3 x 3 matrices to improve the colorimetry of colour cameras is fairly well established in BBC practice. Some problems remain such as the matching of several cameras operating in one studio and the present report investigates theoretically the properties of 2 x 2 matrices operating on linear colour difference signals. These matrices give independent-rotation of two colour axes together with independent changes of saturation: in principle the luminance of colours is not altered.

1. INTRODUCTION

The use of a 3 x 3 matrix operating on linear RGB signals is now well established.[1,2,3] In the case of a three-tube camera, both chromaticity and luminance errors can be corrected to a considerable extent, resulting in colour fidelity of a high order. In the case of a four-tube camera, a 3 x 3 matrix has been used in the colouring channel primarily to correct errors of chromaticity.

Much theoretical and practical effort has been spent in investigating the application of 3 x 3 matrices to specific cameras (not merely to various types of camera but also to various optical-analysis systems when incorporated in one type of camera). Although a high order of colour rendering is frequently achieved, the matching of two cameras of nominally identical specification is sometimes a problem and it is conceivable that final matching of cameras could be achieved by a matrix operation on linear colour-difference signals.†

Since two colour-difference signals (e.g. $R-Y$ and $B-Y$) are sufficient to completely specify the chromaticity of a picture element (provided that the luminance Y is also known), matrix operation on colour-difference signals takes the form of a 2 x 2 matrix. It is not envisaged that this type of matrix would replace the 3 x 3 matrices at present in use but rather that it might be used to supplement them for the purpose of camera matching, for example. For this purpose a matrix based on colour-difference signals should be superior to 'paint pot' controls in that white point balance is independent of the matrix coefficients.

† This concept arose from discussions with Mr. C.B.B. Wood.

2. MATRIX OPERATION ON COLOUR-DIFFERENCE SIGNALS

The form assumed for the 2 x 2 matrix is expressed in Equation (1).

$$\begin{bmatrix}(R-Y)^*\\(B-Y)^*\end{bmatrix} = \begin{bmatrix}a_{11} & a_{12}\\a_{21} & a_{22}\end{bmatrix}\begin{bmatrix}(R-Y)\\(B-Y)\end{bmatrix} \qquad (1)$$

The signals are linear and subject to the usual setting up procedure** and there is, in principle, no restriction on the values of a_{11} to a_{22}. As they are linear colour-difference signals, the luminance is unchanged and the colour balance point is unchanged for any values of a_{11} ... a_{22}. This is in contrast to the 3 x 3 matrix where a subsidiary condition to preserve colour balance is that the sums of the rows must be unity. Hence a 3 x 3 matrix as used with colour cameras has six independent variables. The 2 x 2 matrix has four independent variables.

In practice it is envisaged that the principal coefficients, a_{11} and a_{22} would be adjustable in the range (say) 1·0 ± 0·1 and the subsidiary coefficients a_{12} and a_{21} in the range 0 ± 0·1. The effect of varying the coefficients over

** i.e. $R = G = B = Y = 1$ on white.

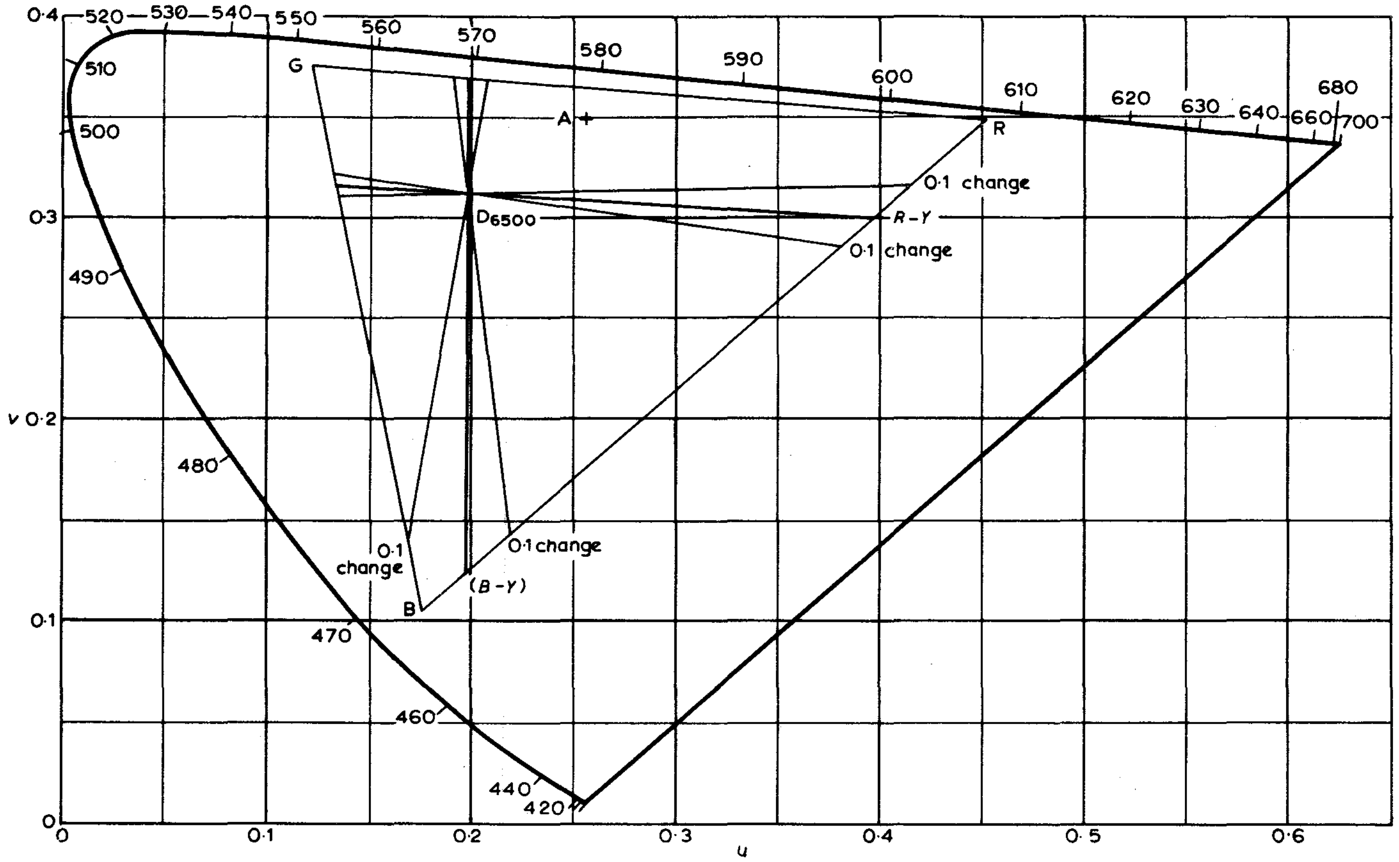

Fig. 1 - R–Y and B–Y axes and the effect of ±0·1 change in the coefficients of a [2x2] matrix controlling these (linear) signals

this range is shown in Fig. 1 which gives the variations in direction of the $(R-Y)$ and $(B-Y)$ axes. With four independent variables, this enables control over direction and magnitude of the two colour-difference signals to be exercised.

3. MATHEMATICAL RELATIONSHIPS

A 2 x 2 matrix operation on linear colour-difference signals must have an equivalent form of 3 x 3 matrix formulation. This can be derived in the following way:

$$\begin{bmatrix} Y \\ R-Y \\ B-Y \end{bmatrix} = \begin{bmatrix} l & m & n \\ 1-l & -m & -n \\ -l & -m & 1-n \end{bmatrix} \begin{bmatrix} R_1 \\ G_1 \\ B_1 \end{bmatrix} \qquad (2)$$

where l, m, n are (nominal) luminosity coefficients.

R_1, G_1, B_1 are the primary linear signals given by a three-tube camera with or without the operation of a 3 x 3 matrix.

$$\begin{bmatrix} Y^* \\ (R-Y)^* \\ (B-Y)^* \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & a_{11} & a_{12} \\ 0 & a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} Y \\ R-Y \\ B-Y \end{bmatrix} \qquad (3)$$

$$\begin{bmatrix} R^* \\ G^* \\ B^* \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & -l/m & -n/m \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} Y^* \\ (R-Y)^* \\ (B-Y)^* \end{bmatrix} \qquad (4)$$

In Equations (2) and (3) the starred values are those given by the matrix 3 which is essentially matrix 1.

If Equations (2), (3) and (4) are multiplied together we obtain:

$$\begin{bmatrix} R^* \\ G^* \\ B^* \end{bmatrix} = \begin{bmatrix} a_{11}+l(1-a_{11}-a_{12}) & m(1-a_{11}-a_{12}) & a_{12}+n(1-a_{11}-a_{12}) \\ l-\frac{l}{m}(a_{11}-la_{11}-la_{12}) - \frac{n}{m}(a_{21}-la_{21}-la_{22}) & m+l(a_{11}+a_{12}) + n(a_{21}+a_{22}) & n-\frac{l}{m}(a_{12}-na_{11}-na_{12}) - \frac{n}{m}(a_{22}-na_{21}-na_{22}) \\ a_{21}+l(1-a_{21}-a_{22}) & m(1-a_{21}-a_{22}) & a_{22}+n(1-a_{21}-a_{22}) \end{bmatrix} \begin{bmatrix} R_1 \\ G_1 \\ B_1 \end{bmatrix} \qquad (5)$$

For the special case $a_{11} = a_{22} = 1$, $a_{21} = a_{12} = 0$, matrix 5 can be shown to become a unit matrix, as it should.

Another special case is the restriction that

$$\begin{aligned} a_{11} + a_{12} &= 1 \\ a_{21} + a_{22} &= 1 \end{aligned} \qquad (6)$$

In this case we have only two independent variables and matrix 5 can be written as

$$\begin{bmatrix} R^* \\ G^* \\ B^* \end{bmatrix} = \begin{bmatrix} a_{11} & 0 & a_{12} \\ \dfrac{l - la_{11} - na_{21}}{m} & 1 & -\dfrac{l - la_{11} - na_{21}}{m} \\ a_{21} & 0 & a_{22} \end{bmatrix} \begin{bmatrix} R_1 \\ G_1 \\ B_1 \end{bmatrix} \qquad (7)$$

From the central column of coefficients it will be deduced that the effective green primary is unchanged.

4. TWO WORKED EXAMPLES

It may be helpful to illustrate the relationships given in Section 3 and the following two examples illustrate (i) a case where Equation (6) holds, (ii) a more general case.

4.1. A Special Case Satisfying Equation (6)

The matrix Equation (8)

$$\begin{bmatrix} (R-Y)^* \\ (B-Y)^* \end{bmatrix} = \begin{bmatrix} 1{\cdot}1 & -0{\cdot}1 \\ 0{\cdot}1 & 0{\cdot}9 \end{bmatrix} \begin{bmatrix} R-Y \\ B-Y \end{bmatrix} \qquad (8)$$

has been evaluated and the directions of the $(R-Y)^*$ and $(B-Y)^*$ axes plotted in Fig. 2 as well as the effective primaries R^* G^* and B^*. To avoid making the example too complicated it is assumed that there is no 3 x 3 matrix treatment on the primary linear signals. Fig. 2 shows that both the $R-Y$ and $B-Y$ axes are rotated in a clockwise direction (compare with Fig. 1 for the natural directions of the $R-Y$ and $B-Y$ axes) but it should be noted that the

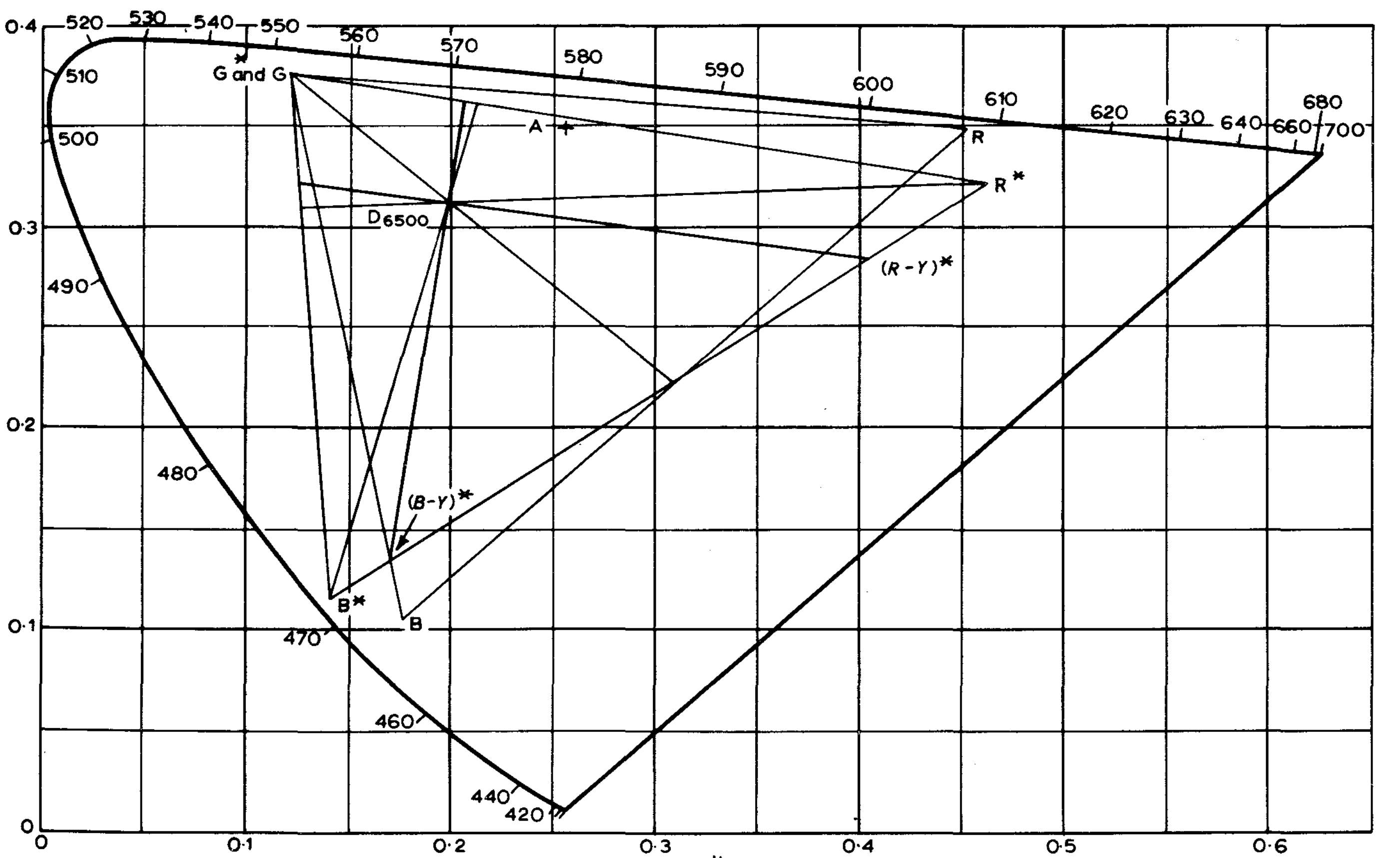

Fig. 2 - Example of matrix operation on $R-Y$ and $B-Y$ linear signals where $a_{11}+a_{12} = a_{21}+a_{22} = 1$

$$\begin{bmatrix} (R-Y)^* \\ (B-Y)^* \end{bmatrix} = \begin{bmatrix} 1{\cdot}1 & -0{\cdot}1 \\ 0{\cdot}1 & 0{\cdot}9 \end{bmatrix} \begin{bmatrix} (R-Y) \\ (B-Y) \end{bmatrix}$$

RGB are display phosphors. R*G*B* are effective primaries. For simplicity there is no 3x3 matrix

green-magenta axis is unchanged, i.e. there is not a more or less uniform rotation of all colour axes in a clockwise direction. Note that the R^*B^* line intersects the RB line at the magenta complementary colour point.

4.2. A Second Numerical Example

The matrix Equation (9) has also been evaluated and is plotted in Fig. 3.

$$\begin{bmatrix}(R-Y)^* \\ (B-Y)^*\end{bmatrix} = \begin{bmatrix}1\cdot2 & -0\cdot1 \\ 0\cdot1 & 1\cdot0\end{bmatrix}\begin{bmatrix}R-Y \\ B-Y\end{bmatrix} \tag{9}$$

This example was chosen to give approximately the same rotations of axes as for matrix 8 but there is a chrominance gain in this example. Although Equation (6) is no longer satisfied, the most general relationship is not shown in matrix 9 because

$$a_{11}+a_{12} = a_{21}+a_{22} \; (= 1\cdot1) \tag{10}$$

As a result of this, the green primary moves in a direction of constant dominant wavelength (from G to G^* on Fig. 3) and the direction of the green-magenta axis is thus unchanged.

Examination of Fig. 3 shows that there is an increase of saturation of almost all colours and a clockwise rotation of hue (e.g. red becomes a purple-red) of many colours although the green-magenta axis exhibits no hue shift (there is however an increase in saturation on this axis).

It should be noted that a change of hue of the effective green primary G^* would be achieved if:

$$a_{11}+a_{12} \neq a_{21}+a_{22} \tag{11}$$

5. CONCLUSIONS

Independent changes of hue and saturation can be achieved along the two colour-difference axes without any change of white point or of the luminance values of the displayed colours. General formulae have been given and two numerical examples are illustrated on the 1960 CIE–UCS diagram.

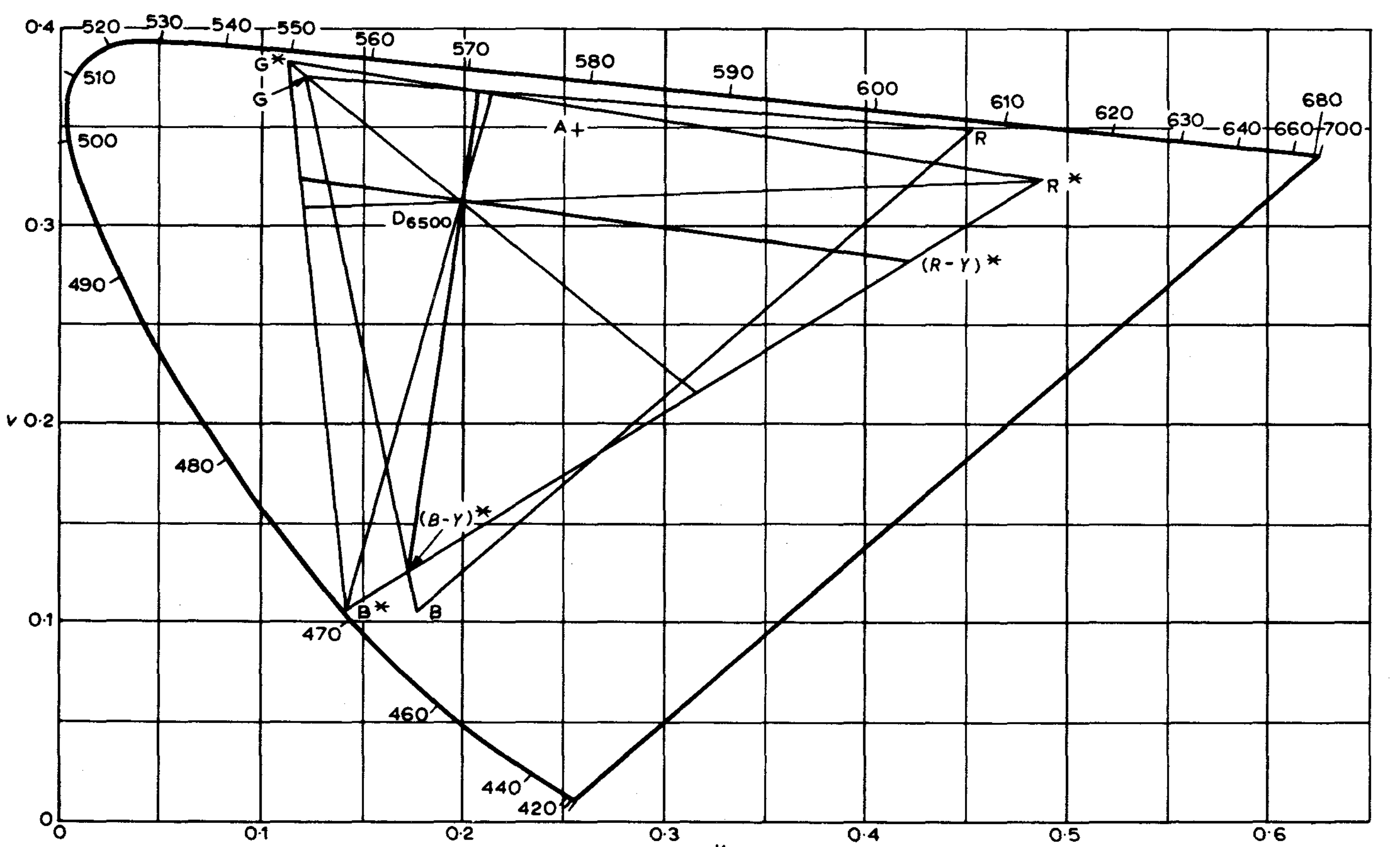

Fig. 3 - Example of matrix operation on linear R−Y and B−Y signals. Unit sum condition not maintained

Direction of green to magenta axis unchanged because sums of both rows were equal: length has expanded

$$\begin{bmatrix}(R-Y)^* \\ (B-Y)^*\end{bmatrix} = \begin{bmatrix}1\cdot2 & -0\cdot1 \\ 0\cdot1 & 1\cdot0\end{bmatrix}\begin{bmatrix}R-Y \\ B-Y\end{bmatrix}$$

6. REFERENCES

1. Use of a linear matrix to modify the colour analysis characteristics of a colour camera. BBC Research Department Report No. T-157, Serial No. 1965/50.

2. Optimum colour analysis characteristics for colour television picture sources with three receptors. BBC Research Department Report No. T-175, Serial No. 1966/56.

3. 3 x 3 matrices for use with plumbicon colour cameras using three tubes. BBC Research Department Report No. PH-15, Serial No. 1967/59.

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